HOMEWORK 7 - ANSWERS TO (MOST) PROBLEMS

PEYAM RYAN TABRIZIAN

Section 3.6: Derivatives of logarithmic functions

3.6.13. $g'(x) = \frac{1}{x\sqrt{x^2-1}}\left(\sqrt{x^2-1} + \frac{x^2}{\sqrt{x^2-1}}\right)$

3.6.21. $y' = 2\log_{10}(\sqrt{x}) + 2x\frac{1}{\ln(10)\sqrt{x}} \times \frac{1}{2\sqrt{x}} = 2\log_{10}(\sqrt{x}) + \frac{1}{\ln(10)}$

3.6.39. $y' = \frac{\sin^2(x)\tan^4(x)}{(x^2+1)^2}\left(2\frac{\cos(x)}{\sin(x)} + 4\frac{\sec^2(x)}{\tan(x)} - \frac{x}{x^2+1}\right)$

Section 3.7: Rates of change in the natural and social sciences

3.7.4.

(a) $f'(t) = e^{-\frac{t}{2}} - \frac{t}{2}e^{-\frac{t}{2}} = e^{-\frac{t}{2}}\left(1 - \frac{t}{2}\right)$

(b) $f'(3) = e^{-\frac{3}{2}}\left(-\frac{1}{2}\right)$

(c) $t = 2$

(d) When $t < 2$

(e) $f(2) - f(0) + f(2) - f(8) = 2e^{-1} - 0 + 2e^{-1} - 8e^{-4} = 4e^{-1} - 8e^{-4}$

(f) The particle is moving to the right between $t = 0$ and $t = 2$, and then to the left from $t = 2$ to $t = 8$.

(g) $f''(t) = -\frac{1}{2}e^{-\frac{t}{2}}\left(1 - \frac{t}{2}\right) + e^{-\frac{t}{2}}\left(-\frac{1}{2}\right) = e^{-\frac{t}{2}}\left(-\frac{1}{2} + \frac{t}{4} - \frac{1}{2}\right) = e^{-\frac{t}{2}}\left(\frac{t}{4} - 1\right)$; $f''(3) = e^{-\frac{3}{2}}\left(-\frac{1}{4}\right)$

(h) Use a calculator

(i) Speeding up when $f''(t) > 0$ and $f'(t) > 0$ or when $f''(t) < 0$ and $f'(t) < 0$. But solving those equations reveals that **none** of the two situations can happen! Hence the particle is constantly slowing down!

3.7.10.

(a) First solve for $v(t) = 0$, where $v(t) = \frac{ds}{dt} = 80 - 32t$, you get $\boxed{t = \frac{80}{32} = \frac{5}{2}}$. So the **maximum height** s^* is $s^* = s(\frac{5}{2}) = 200 - 100 = 100$

(b) To find the time t when the ball is 96ft above the ground, we need to solve the equation $s(t) = 96$, and you get $\boxed{t = 2, 3}$, whence $\boxed{v(2) = 80 - 32 \cdot 2 = 16\frac{ft}{s}}$ and $\boxed{v(3) = 80 - 32 \cdot 3 = -16\frac{ft}{s}}$

Date: Friday, March 11th, 2011.

3.7.17. $f'(x) = 6x =$ linear density at x. $f'(1) = 6$, $f'(2) = 12$, $f'(3) = 18$. The density is highest at 3 and lowest at 1.

3.7.24. See attached document 'Solution to 3.7.24'. You should get $b = 6$, $a = 140$, and the population goes to $a = 140$ (because the denominator goes to 1 as t goes to ∞)

3.7.29.

(a) $C'(x) = 12 - 0.2x + 0.0015x^2$
(b) $C'(200) = 32$; The cost of producing one more yard of a fabric once 200 yards have been produced
(c) $C(201) - C(200) = 32.2005$, which is pretty close to $C'(200)$

Section 3.8: Exponential growth and decay

3.8.4.

(a) $y(0) = C = 120$
(b) $y(t) = 120e^{\frac{\ln(125)}{6}t} = 120(125)^{\frac{t}{6}} = 120\left(5^{\frac{t}{2}}\right)$
(c) $y(5) = 120 \times 5^{\frac{5}{2}} \approx 6708$
(d) $y'(5) = Ky(5) = \frac{\ln(125)}{6} \times 120 \times 5^{\frac{5}{2}} \approx 5398$
(e) $t = 2\frac{\ln\left(\frac{5000}{3}\right)}{\ln(5)} \approx 9.21$

3.8.9.

(a) $y(t) = 100e^{\ln\left(\frac{1}{2}\right)\frac{t}{30}} = 100\left(\frac{1}{2}\right)^{\frac{t}{30}}$
(b) $y(100) = 100\left(\frac{1}{2}\right)^{\frac{100}{30}} \approx 9.92$
(c) $t = 30\frac{\ln\left(\frac{1}{100}\right)}{\ln\left(\frac{1}{2}\right)} \approx 199.3$

3.8.11. We know $Ce^{5730K} = \frac{C}{2}$ and $Ce^{Kt} = 0.74C$, and we need to solve for t. First of all, the first equation gives $e^{5730K} = \frac{1}{2}$, so $K = \frac{\ln(0.5)}{5730} \approx -0.000121$, and from the second equation, we get $e^{Kt} = 0.74$, so $Kt = \ln(0.74)$, so $t = \frac{\ln(0.74)}{\frac{\ln(0.5)}{5730}} \approx 2489$

3.8.19.

(a) (i) $3000\left(1 + \frac{0.05}{1}\right)^{(1)(5)} \approx 3828$
(ii) $3000\left(1 + \frac{0.05}{2}\right)^{(2)(5)} \approx 3840$
(iii) $3000\left(1 + \frac{0.05}{12}\right)^{(12)(5)} \approx 3850$
(iv) $3000\left(1 + \frac{0.05}{52}\right)^{(52)(5)} \approx 3851.61$
(v) $3000\left(1 + \frac{0.05}{365}\right)^{(365)(5)} \approx 3852.01$
(vi) $3000e^{0.05(5)} \approx 3852.08$
(b) $A' = 0.05A$, $A(0) = 3000$

Section 3.9: Related rates

3.9.5. $\boxed{\frac{dh}{dt} = \frac{3}{25\pi}}$ (Use $V = \pi r^2 h$)

3.9.13. $\boxed{\frac{d(x+y)}{dt} = \frac{25}{3}}$ (use the law of similar triangles to get $\frac{x}{x+y} = \frac{3}{5}$)

3.9.15. $\boxed{\frac{dD}{dt} = 65mph}$ (use the pythagorean theorem to conclude $D^2 = x^2 + y^2$)

3.9.27. $\boxed{\frac{dh}{dt} = \frac{6}{5\pi}}$ (use the fact that $V = \frac{\pi}{12}h^3$ because $h = \frac{r}{2}$)

3.9.38. See attached document 'Solution to 3.9.38'. Use the definition of $\tan(\theta)$, $\boxed{-\frac{80\pi}{3}}$

3.9.43. See attached document 'Solution to 3.9.43'. You should get $\boxed{x' = \frac{7}{4}\sqrt{15}}$